Statistical Clock Tree Routing for Robustness to Process Variations

Uday Padmanabhan\(^1\), Janet M. Wang\(^1\) and Jiang Hu\(^2\)

\(^1\)University of Arizona at Tucson
\(^2\)Texas A&M University
Outline

- Previous research
- Motivation of this project
- Variation aware delay model
- Statistical centering algorithm
- Examples
- Conclusion
Clock Tree Routing

Merging segment

Permissible range

PDF

Skew

Permissible range

PDF

Skew
Previous Research

- Layout embedding method to achieve zero skew
- Deferred Merging Embedding technique (DME)
- Nearest neighbor based abstract tree method (NNA)
- Bounded skew tree (BST) algorithm
- Minimal Skew Violations (MinSV)
Permissible Range

Align the mean to the center
Skew and Permissible Range

Center of worst skew range

Skew permissible range

Median

L U

Center of worst skew range

Skew permissible range

Median

L U
Basic Concepts

Variance

Skewness

Kurtosis

Positively Skewed Distribution
Existing Issues

- Most of the research works are for deterministic clock tree routing
- Some research works consider corner cases
  
  not work well with statistical clock skew
Possible Strategies

- Skew can be non-Gaussian distributed: asymmetric
- Only mean and variance are not enough
- Our methodology:
  - include high order moments
  - center measures include: Mode, Median and Mean
A Task List

- Variation aware delay model
- Parameter reduction based on ANOVA and OPCA
- Choose among Mean, Median and Mode as Central Measures
- Select wire length based on Central Measures
- Variation aware abstract tree topology
Delay model

\[ d_{FEDk} = A r_d c_d \sum_{i \in T} l_i w_i + B r_d c_f \sum_{i \in T} l_i + C r_d \sum_{j \in S} cL_j \]

\[ + D \rho c_a \sum_{i \in P_k} \frac{l_i}{w_i t_i} \left( \frac{l_i w_i}{2} + \sum_{j \in E_i} l_j w_j \right) \]

\[ + E \rho c_f \sum_{i \in P_k} \frac{l_i}{w_i t_i} \left( \frac{l_i}{2} + \sum_{j \in E_i} l_j \right) \]

\[ + F \rho \sum_{i \in P_k} \frac{l_i}{w_i t_i} \left( \sum_{j \in S_i} cL_j \right) \]
Delay Mean and variance

First Order Estimates

\[
\mu_T = d_{FED} \bigg|_{w=\mu_w, t=\mu_t, \rho = \rho_d, c_L = \mu_c} + \\
\frac{1}{2} \left[ \left( E \frac{\rho c_f l^2}{t} + 2 F \frac{\rho l c_L}{t} \right) \frac{1}{\mu_w^3} \right. \\
\left. + (D \rho c_a l^2 + E \frac{\rho c_f l^2}{w} + 2 F \frac{\rho l c_L}{w}) \frac{1}{\mu_t^3} \right]
\]

\[
\sigma_T^2 \square \left( \frac{\partial d_{FED}}{\partial r_d} \right)^2 \sigma_{r_d}^2 + \left( \frac{\partial d_{FED}}{\partial c_L} \right)^2 \sigma_{c_L}^2 \\
+ \left( \frac{\partial d_{FED}}{\partial w} \right)^2 \sigma_w^2 + \left( \frac{\partial d_{FED}}{\partial t} \right)^2 \sigma_t^2
\]
Delay Mean and Variance

\[ \mu_{Til} = \sum_{k \in P_i} \mu_{Tk} \]
\[ \sigma_{Til}^2 = \sum_{k \in P_i} \sigma_{Tk}^2 \]

\[ \mu_{Slr} = \mu_{Til} - \mu_{Tjr} \]
\[ \sigma_{Slr}^2 = \sigma_{Til}^2 + \sigma_{Tjr}^2 \]
Delay Median

Edge Delay Median:

\[ M_T = D \frac{\rho c_a l^2}{2M_t} + \frac{E \rho c_f l^2}{2M_t M_w} + \frac{F \rho M_{c_L}}{M_t M_w} \]

Path Delay Median:

\[ M_{Til} = \sum_{k \in P_{il}} M_{Tk} \]

Skew Median:

\[ M_{Slr} = M_{Til} - M_{Tjr} \]
Delay Mode

Matching the delay mode by Weibull function:

\[ f_{WB}(x) = \alpha \beta^{-\alpha} x^{\alpha-1} e^{-(x/\beta)^{\alpha}} \]

\[ \mu_{WB} = \beta \Gamma(1 + \theta) \]

\[ \sigma_{WB}^2 = \beta^2 \left( \Gamma(1 + 2\theta) - \Gamma^2(1 + \theta) \right) \]

\[ \frac{\Gamma(1 + 2\theta)}{\Gamma^2(1 + \theta)} = \frac{\sigma_{Slr}^2 + \mu_{Slr}^2}{\mu_{Slr}^2} \quad \beta = -m_1/\Gamma(1 + \theta). \]

\[ \text{Mode} = \frac{\alpha - 1}{\sqrt{\beta^{-\alpha} \alpha}} \]
ANOVA

\[ \mu_{P_{s_1,n_7}}(w_1,t_1,w_2,t_2) = \mu_{s_1,n_5}(w_1,t_1) + \mu_{n_5,n_7}(w_2,t_2) \]
CASE I. $|\xi| \leq \varepsilon$, $\forall |PR_{lr}|$. The distribution is symmetric or almost symmetric. Therefore, $CM_{Slr} = \mu_{Slr}$.

CASE II. $|\xi| > \varepsilon$, $|PR_{lr}| > 5\sigma$. The distribution can be easily fit within the permissible range by aligning the mean. Therefore, $CM_{Slr} = \mu_{Slr}$. Choosing the median or mode might lead to an excessive part of the distribution lying outside the permissible range.

CASE III. $|\xi| > \varepsilon$, $2\sigma < |PR_{lr}| < 5\sigma$. The permissible range is not large enough to fit the entire distribution and is not extremely narrow either. As shown in Figure 6(c), $CM_{Slr} = M_{Slr}$ is the best choice in this case.

CASE IV. $|\xi| > \varepsilon$, $|PR_{lr}| < 2\sigma$. The permissible range represents a very stringent constraint. Based on the previous discussion, we choose $CM_{Slr} = Mode_{Slr}$ to maximize the area within bounds.
Statistical Centering Based Layout Embedding

(a) Permissible Range = 3σ
(b) Permissible Range = 6σ
(c) Permissible Range = 3σ
(d) Permissible Range = 1σ
Mean, Median and Mode based design

\[ q_{ij} = A_s l_i^2 + B_s l_i - C_s \]

where

\[
A_s = \frac{\rho}{2} \left[ c_a \left( \frac{D_i}{t_i} - \frac{D_j}{t_j} \right) + c_f \left( \frac{E_i}{w_i t_i} - \frac{E_j}{w_j t_j} \right) \right] \\
B_s = \rho \left[ d_{ij} \left( \frac{D_j c_a}{t_j} + \frac{E_j c_f}{w_j t_j} \right) + \frac{F_i c_{Li}}{w_i t_i} + \frac{F_j c_{Lj}}{w_j t_j} \right] \\
C_s = F_j \rho c_{Lj} d_{ij} \frac{1}{w_j t_j}
\]
Variation Aware Abstract Tree Topology

- Extend the Nearest Neighbor Algorithm (NNA)
- Length is positive dependent on the distance between nodes

\[ \partial l = f(d_{ij}) \partial d_{ij} \]

- The minimal composite distance

\[ d'_{ij} = d^2_{ij} \left( \lambda + \left| \frac{1}{w_i} - \frac{1}{w_j} \right| \right) \]
Process variation model
## Experimental Results

<table>
<thead>
<tr>
<th></th>
<th>Profile1</th>
<th></th>
<th>#skew violations</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>wirelength ((\mu m))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NNA+MinSV</td>
<td>NEW</td>
<td>Imprv</td>
<td>NNA+MinSV</td>
<td>NEW</td>
<td>Imprv</td>
</tr>
<tr>
<td>r1</td>
<td>137848</td>
<td>127364</td>
<td>8%</td>
<td>74</td>
<td>61</td>
<td>17%</td>
</tr>
<tr>
<td>r2</td>
<td>292006</td>
<td>257338</td>
<td>12%</td>
<td>218</td>
<td>140</td>
<td>36%</td>
</tr>
<tr>
<td>r3</td>
<td>339958</td>
<td>322035</td>
<td>5%</td>
<td>246</td>
<td>194</td>
<td>21%</td>
</tr>
<tr>
<td>r4</td>
<td>695116</td>
<td>657460</td>
<td>5%</td>
<td>358</td>
<td>287</td>
<td>20%</td>
</tr>
<tr>
<td>r5</td>
<td>1034488</td>
<td>983639</td>
<td>5%</td>
<td>501</td>
<td>363</td>
<td>28%</td>
</tr>
<tr>
<td>Prim1</td>
<td>134045</td>
<td>131013</td>
<td>2%</td>
<td>42</td>
<td>37</td>
<td>12%</td>
</tr>
<tr>
<td>Prim2</td>
<td>352054</td>
<td>308380</td>
<td>12%</td>
<td>155</td>
<td>97</td>
<td>37%</td>
</tr>
<tr>
<td>s1423</td>
<td>110625</td>
<td>104935</td>
<td>5%</td>
<td>26</td>
<td>20</td>
<td>23%</td>
</tr>
<tr>
<td>s5378</td>
<td>221017</td>
<td>169052</td>
<td>23%</td>
<td>75</td>
<td>56</td>
<td>25%</td>
</tr>
<tr>
<td>s15850</td>
<td>446656</td>
<td>431137</td>
<td>3%</td>
<td>226</td>
<td>199</td>
<td>12%</td>
</tr>
</tbody>
</table>
## Experimental Results

<table>
<thead>
<tr>
<th>Profile2</th>
<th>wirelength ($\mu$m)</th>
<th>#skew violations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NNA+MinSV</td>
<td>NEW</td>
</tr>
<tr>
<td>----------</td>
<td>-----------</td>
<td>-----</td>
</tr>
<tr>
<td>135012</td>
<td>131155</td>
<td>3%</td>
</tr>
<tr>
<td>286685</td>
<td>262892</td>
<td>8%</td>
</tr>
<tr>
<td>347469</td>
<td>329361</td>
<td>5%</td>
</tr>
<tr>
<td>691496</td>
<td>664143</td>
<td>4%</td>
</tr>
<tr>
<td>1036414</td>
<td>986358</td>
<td>5%</td>
</tr>
<tr>
<td>135352</td>
<td>132238</td>
<td>2%</td>
</tr>
<tr>
<td>328610</td>
<td>312532</td>
<td>5%</td>
</tr>
<tr>
<td>109588</td>
<td>108727</td>
<td>0%</td>
</tr>
<tr>
<td>174745</td>
<td>170404</td>
<td>2%</td>
</tr>
<tr>
<td>445104</td>
<td>437260</td>
<td>2%</td>
</tr>
</tbody>
</table>
Experimental Results
## Run Time Comparison

<table>
<thead>
<tr>
<th></th>
<th>Sinks</th>
<th>CPU(s)</th>
<th>NNA+MinSV</th>
<th>NEW</th>
</tr>
</thead>
<tbody>
<tr>
<td>r1</td>
<td>267</td>
<td>1.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>r2</td>
<td>598</td>
<td>2.3</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>r3</td>
<td>862</td>
<td>3.3</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>r4</td>
<td>1903</td>
<td>10.1</td>
<td>10.8</td>
<td></td>
</tr>
<tr>
<td>r5</td>
<td>3101</td>
<td>24.5</td>
<td>26.1</td>
<td></td>
</tr>
<tr>
<td>Prim1</td>
<td>269</td>
<td>1.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Prim2</td>
<td>603</td>
<td>2.2</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>s1423</td>
<td>74</td>
<td>1.3</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>s5378</td>
<td>179</td>
<td>1.4</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>s15850</td>
<td>597</td>
<td>2.3</td>
<td>2.3</td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

- A Statistical centering approach
- A Fitted Elmore delay model with analysis of variance and principle component analysis
- A topology generation algorithm which takes the width and thickness into consideration
Thank you