Estimating Routing Congestion using Probabilistic Analysis

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Congestion Problem

- Wiring congestion is a key metric to predict routability & performance of a design
  - A good timing result is useful only if routing can be completed
  - Congestion increases delay uncertainty
  - Congestion deteriorate signal integrity

- Congestion optimization must be performed early in design cycles
  - Synthesis adds many new gates to the design increasing cell density and wiring demand
  - Timing-driven placement has to keep critical gates closer
  - Users would like to close timing in same or smaller floorplans

- Requires a fast and accurate estimator
Physical Synthesis Flow

- RTL, Constraints
- Resource sharing/allocation
- Implementation selection
- Logic structuring
- Technology mapping
- Gate-level optimization
- Optimized netlist
- Placement
- Floorplan
- Congestion estimation and optimization
Summary of Our Contributions

- Proposed a probabilistic based congestion estimator
- Based on supply and demand analysis of routing resources
- Independent of implementation details of downstream routing algorithms
- Blockage aware
- Fast and yet accurate
- Congestion optimization based on this algorithm demonstrates its effectiveness
Congestion Analysis

- Divide the core area of the design into congestion grids
- Analyze the supply and demand for routing resources in each congestion grid
Routing Supply of a Grid

- $N_h$: number of horizontal routing layers
- $N_v$: number of vertical routing layers
- $L_{hi}$: minimum pitch for the $i^{th}$ horizontal layer
- $L_{vi}$: minimum pitch for the $i^{th}$ vertical layer

**Horizontal Capacity**

$$\text{horizontal } \text{capacity} = H \times \sum_{i=1}^{N_h} \left( \frac{1}{L_h^i} \right)$$

**Vertical Capacity**

$$\text{vertical } \text{capacity} = W \times \sum_{i=1}^{N_v} \left( \frac{1}{L_v^i} \right)$$
Routing Demand of a Grid

- **Empirical model**
  - Design dependent
  - Hard to correlate to the real congestion

- **Global router**
  - Slow
  - Introduce problems for 3rd party routers

- **Probabilistic analysis**
  - Fast
  - Router independent
An Example

Estimate a 2-pin nets covering a 3×3 mesh

Total number of possible routes: 6

Horizontal usage = 0.5 + 0.5 + 1.0 = 2.0
Vertical usage = 1.0 + 1.0 = 2.0

\[
\frac{1}{6} \times \begin{bmatrix}
1 & 1 & 2 & 2 & 3 & 3 \\
2 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 2 & 2 & 1 & 1
\end{bmatrix}
\]
Number of Possible Routes

For a 2-pin net covering a \(m \times n\) mesh, denote the number of possible routes as \(F(m, n)\):

\[
F(m, n) = F(m-1, n) + F(m, n-1)
\]

\[
F(m, n) = F(n, m)
\]

\[
F(m, n) = \begin{cases} 
1 & n = 1 \\
m & n = 2 \\
\sum_{i_1=1}^{m} \sum_{i_2=1}^{i_{n-2}} \cdots \sum_{i_1=1}^{i_{n-3}} i_{n-2} & n \geq 3
\end{cases}
\]
$F(m,n)$ up to $10 \times 10$

\[
\begin{bmatrix}
1 & 10 & 55 & 220 & 715 & 2002 & 5005 & 11440 & 24310 & 48620 \\
1 & 9 & 45 & 165 & 495 & 1287 & 3003 & 6435 & 12870 & 24310 \\
1 & 8 & 36 & 120 & 330 & 792 & 1716 & 3432 & 6435 & 11440 \\
1 & 7 & 28 & 84 & 210 & 462 & 924 & 1716 & 3003 & 5005 \\
1 & 6 & 21 & 56 & 126 & 252 & 462 & 792 & 1287 & 2002 \\
1 & 5 & 15 & 35 & 70 & 126 & 210 & 330 & 495 & 715 \\
1 & 4 & 10 & 20 & 35 & 56 & 84 & 120 & 165 & 220 \\
1 & 3 & 6 & 10 & 15 & 21 & 28 & 36 & 45 & 55 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
\]
## Probabilistic Usage Matrix

\[
P(m,n) = \begin{bmatrix}
(P_x(m,1) & P_y(m,1)) & \cdots & (P_x(m,n) & P_y(m,n)) \\
\vdots & \ddots & \vdots & \vdots \\
(P_x(1,1) & P_y(1,1)) & \cdots & (P_x(1,n) & P_y(1,n))
\end{bmatrix}
\]

\[
P_{x|y}(i,j) = P_{x|y}(m-i+1, n-j+1)
\]

\[
\sum_{j=1}^{n} P_y(i,j) = 1 \quad \forall i
\]

\[
\sum_{i=1}^{m} P_x(i,j) = 1 \quad \forall j
\]
Computing the Probabilities

$$P_x(i, j) = \frac{1}{F(m, n)} \times \begin{cases} 
F(m, n-1) & \text{case a: } i = 1, j = 1 \\
1 & \text{case b: } i = 1, j = n \\
F(m-i+1, n-1) & \text{case c: } 1 < i < m, j = 1 \\
\frac{F(m,n-j+1)+F(m,n-j)}{2} & \text{case d: } i = 1, 1 < j < n \\
\end{cases}$$

$$P_y(i, j) = \frac{1}{F(m, n)} \times \begin{cases} 
F(m-1, n) & \text{case a: } i = 1, j = 1 \\
1 & \text{case b: } i = 1, j = n \\
\frac{F(m-i+1,n)+F(m-i,n)}{2} & \text{case c: } 1 < i < m, j = 1 \\
F(m-1,n-j+1) & \text{case d: } i = 1, 1 < j < n \\
\frac{F(i,j)F(m-i,n-j+1)+F(i-1,j)F(m-i+1,n-j+1)}{2} & \end{cases}$$
Off-grid Pins
Horizontal Usage with Off-grid Pins

\[ P_x (i, j) = \frac{1}{F(m,n)} \times \begin{cases} 
F (m, n - 1) \times \frac{d_{x1}}{W} & \text{case } a: i = 1, j = 1 \\
F (m, n - 1) \times \frac{d_{x2}}{W} & \text{case } \tilde{a}: i = m, j = n \\
\frac{d_{x2}}{W} & \text{case } b: i = 1, j = n \\
\frac{d_{x1}}{W} & \text{case } \tilde{b}: i = m, j = 1 \\
F (m - i + 1, n - 1) \times \frac{d_{x1}}{W} & \text{case } c: 1 < i < m, j = 1 \\
F (i, n - 1) \times \frac{d_{x2}}{W} & \text{case } \tilde{c}: 1 < i < m, j = n \\
\frac{F (m, n - j + 1) + F (m, n - j)}{2} & \text{case } d: i = 1, 1 < j < n \\
\frac{F (m, j) + F (m, j - 1)}{2} & \text{case } \tilde{d}: i = m, 1 < j < n \\
\frac{F (i, j) F (m - i + 1, n - j) + F (i, j - 1) F (m - i + 1, n - j + 1)}{2} & \end{cases} \]
Vertical Usage with Off-grid Pins

\[
P_y(i, j) = \frac{1}{F(m, n)} \times \begin{cases} 
F(m-1, n) \times \frac{d_{y1}}{H} & \text{case } a: i=1, j=1 \\
F(m-1, n) \times \frac{d_{y2}}{H} & \text{case } \tilde{a}: i=m, j=n \\
d_{y1} \quad \frac{1}{H} & \text{case } b: i=1, j=n \\
d_{y2} \quad \frac{1}{H} & \text{case } \tilde{b}: i=m, j=1 \\
F(m-i+1, n) + F(m-i, n) \quad 2 & \text{case } c: 1<i<m, j=1 \\
F(i, n) + F(i-1, n) \quad 2 & \text{case } \tilde{c}: 1<i<m, j=n \\
F(m-1, n-j+1) \times \frac{d_{y1}}{H} & \text{case } d: i=1, 1<j<n \\
F(m-1, j) \times \frac{d_{y2}}{H} & \text{case } \tilde{d}: i=m, 1<j<n \\
F(i, j) F(m-i, n-j+1) + F(i-1, j) F(m-i+1, n-j+1) \quad 2 & \text{case } \tilde{d}: i=m, 1<j<n \\
\end{cases}
\]
Track Usages for a $5 \times 5$ Mesh

- Darker color represents higher probability of usages
Multi-pin Nets

- MST based algorithm in early stages
- RST based algorithm in later stages
Simple Routing Blockages

Distribute the usage of a blocked grid to its neighboring grids based on the distance $d$ and number of unblocked neighbors $n$:

$$w = 2^{-d} \times n$$
Line Blockages

- Extend the bounding box of the net
Adjacent Blockages

• Find the bounding box of the blockages
Complex Blockages

- Maze router needed
- How often do you see this?
## Runtime for two Testcases

<table>
<thead>
<tr>
<th></th>
<th># of instances</th>
<th># of nets</th>
<th>CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design1</td>
<td>316K</td>
<td>332K</td>
<td>70s</td>
</tr>
<tr>
<td>Design2</td>
<td>347K</td>
<td>374K</td>
<td>110s</td>
</tr>
</tbody>
</table>
Testcase I: Congestion Correlation

Estimated congestion  Post-route congestion
Testcase II: Congestion Correlation

Estimated congestion

Post-route congestion
Testcase I: Congestion Removal

Congestion without optimization

Congestion with optimization
Testcase II: Congestion Removal

Congestion without optimization

Congestion with optimization
Conclusion

- Timing closure for complex designs requires fast and accurate congestion estimators
- Probabilistic congestion estimation is a key technology in physical synthesis
- Congestion optimization based on this algorithm demonstrates its effectiveness