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Closing the Smoothness and Uniformity Gap in Area Fill Synthesis

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http://vlsicad.ucsd.edu
Outline

- *Layout Density Control for CMP*
- Our Contributions
- Layout Density Analysis
- Local Density Variation
- Summary and Future Research
CMP and Interlevel Dielectric Thickness

- Chemical-Mechanical Planarization (CMP)
  = wafer surface planarization
- Uneven features cause polishing pad to deform

- Interlevel-dielectric (ILD) thickness $\approx$ feature density
- Insert dummy features to decrease variation
Objectives of Density Control

• Objective for Manufacture = Min-Var
  minimize window density variation
  subject to upper bound on window density

• Objective for Design = Min-Fill
  minimize total amount of filling
  subject to fixed density variation
Filling Problem

- Given
  - rule-correct layout in $n \times n$ region
  - window size = $w \times w$
  - window density upper bound $U$

- Fill layout with Min-Var or Min-Fill objective such that NO fill is added
  - within buffer distance $B$ of any layout feature
  - into any overfilled window that has density $\geq U$
Fixed-Dissection Regime

- Monitor only fixed set of $w \times w$ windows
  - “offset” = $w/r$ (example shown: $w = 4$, $r = 4$)
- Partition $n \times n$ layout into $nr/w \times nr/w$ fixed dissections
- Each $w \times w$ window is partitioned into $r^2$ tiles
Previous Works

- Kahng et al.
  - first formulation for fill problem
  - layout density analysis algorithms
  - first LP based approach for Min-Var objective
  - Monte-Carlo/Greedy
  - iterated Monte-Carlo/Greedy
  - hierarchical fill problem

- Wong et al.
  - Min-Fill objective
  - dual-material fill problem
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Our Contributions

• Smoothness gap in existing fill methods
  ○ large difference between fixed-dissection and floating
    window density analysis
  ○ fill result will not satisfy the given upper bounds

• New smoothness criteria: local uniformity
  ○ three new relevant Lipschitz-like definitions of local density
    variation are proposed
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Oxide CMP Pattern Dependent Model

- Removal rate inversely proportional to density
  \[
  \frac{dz}{dt} = - \frac{K}{\rho(x, y)}
  \]

- Density assumed constant (equal to pattern) until local step has been removed:
  \[
  \rho(x, y, z) = \begin{cases}
    \rho_0(x, y) & z > z_0 - z_1 \\
    1 & z < z_0 - z_1
  \end{cases}
  \]

- Final Oxide thickness related to local pattern density
  \[
  z = \begin{cases}
    z_0 - \left( \frac{K_i t}{\rho(x, y)} \right) & t < (\rho_0 z_1) / K_i \\
    z_0 - z_1 - K_i t + \rho_0(x, y) z_1 & t > (\rho_0 z_1) / K_i
  \end{cases}
  \]

  pattern density \( \rho_0(x, y) \) is crucial element of the model.

\( z = \) final oxide thickness over metal features
\( K_i = \) blanket oxide removal rate
\( t = \) polish time
\( \rho_0 = \) local pattern density

(Stine et al. 1997)
Layout Density Models

- **Spatial Density Model**
  \[ \text{window density} \approx \text{sum of tiles feature area} \]

- **Effective Density Model** (more accurate)
  \[ \text{window density} \approx \text{weighted sum of tiles' feature area} \]
  - weights decrease from window center to boundaries
The Smoothness Gap

- Fixed-dissection analysis \(\neq\) floating window analysis

- Fill result will not satisfy the given bounds

- Despite this gap observed in 1998, all published filling methods fail to consider this smoothness gap
Accurate Layout Density Analysis

- Optimal extremal-density analysis with complexity
  \[\Rightarrow \text{inefficient}\]

- Multi-level density analysis algorithm
  
  An arbitrary floating window contains a shrunk window and is covered by a bloated window of fixed r-dissection

![Diagram showing fixed dissection window, arbitrary window W, shrunk fixed dissection window, bloated fixed dissection window, and tile.](image)
Multi-Level Density Analysis

- Make a list $ActiveTiles$ of all tiles
- $Accuracy = \infty, r = 1$
- WHILE $Accuracy > 1 + 2\varepsilon$ DO
  - find all rectangles in tiles from $ActiveTiles$
  - add windows consisting of $ActiveTiles$ to $WINDOWS$
  - $Max =$ maximum area of window with tiles from $ActiveTiles$
  - $BloatMax =$ maximum area of bloated window with tiles from $ActiveTiles$
  - FOR each tile $T$ from $ActiveTiles$ which do not belong to any bloated window of area $> Max$ DO
    - remove $T$ from $ActiveTiles$
  - replace in $ActiveTiles$ each tile with four of its subtiles
  - $Accuracy = BloatMax/Max, r = 2r$
- Output max window density $= (Max + BloatMax)/(2*w^2)$
Multi-level Density Analysis on Effective Density Model

- Assume that the effective density is calculated with the value of r-dissection used in filling process
- The window phase-shift will be smaller
- Each cell on the left side has the same dimension as the one on right side
# Accurate Analysis of Existing Methods

Multi-level density analysis on results from existing fixed-dissection filling methods

- The window density variation and violation of the maximum window density in fixed-dissection filling are underestimated.
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Local Density Variation

- Global density variation does not take into account that CMP polishing pad can adjust the pressure and rotation speed according to pattern distribution.

- The influence of density variation between far-apart regions can be reduced by pressure adjustment.

- Only a significant density variation between neighboring windows will complicate polishing pad control and cause either dishing or underpolishing.

Density variations between neighboring windows.
Lipschitz-like Definitions

- Local density variation definitions
  - **Type I:**
    - max density variation of every $r$ neighboring windows in each row of the fixed-dissection
    - The polishing pad move along window rows and only overlapping windows in the same row are neighbored
  - **Type II:**
    - max density variation of every cluster of windows which cover one tile
    - The polishing pad touch all overlapping windows simultaneously
  - **Type III:**
    - max density variation of every cluster of windows which cover $\frac{r}{2} \times \frac{r}{2}$ tiles
    - The polishing pad is moving slowly and touching overlapping windows simultaneously
Behaviors of Existing Methods on Smoothness Objectives

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Comparison among the behaviors of existing methods w.r.t Lipschitz objectives

- The solution with the best Min-Var objective value does not always have the best value in terms of “smoothness” objectives
Linear Programming Formulations

- **Lipschitz Type I**
  
  \[
P_{ij} \geq 0 \quad i, j = 0, ..., nr / w - 1
  \]
  
  \[
P_{ij} \leq \text{slack} \ (T_{ij}) \quad i, j = 0, ..., nr / w - 1
  \]
  
  \[
  \sum_{s=i}^{i+r-1} \sum_{t=j}^{j+r-1} p_{st} \leq \alpha_{ij} (U \cdot w^2 - \text{area}_{ij}) \quad i, j = 0, ..., nr / w - 1
  \]
  
  \[
  W_{ij} - W_{ik} \leq L \quad i, j, k = 0, ..., nr / w - 1
  \]

  **here,** \[
  W_{ij} = \sum_{s=i}^{i+r-1} \sum_{t=j}^{j+r-1} \text{area} (T_{st}) + \sum_{s=j}^{i+r-1} \sum_{t=j}^{j+r-1} p_{st}
  \]

- **Lipschitz Type II**

  \[
  \min \ Den(i, j) \leq W_{lm} \leq \max \ Den(i, j) \quad i, j, k = 0, ..., \frac{nr}{w} - 1
  \]

  \[
  \max \ Den(i, j) - \min \ Den(i, j) \leq L \quad l(m) = i(j) - r, ..., i(j) + r
  \]
Linear Programming Formulations

- **Lipschitz Type III**

\[
\begin{align*}
\min Den(i, j) \leq W_{lm} \leq \max Den(i, j) & \quad i, j, k = 0, \ldots, \frac{nr}{w} - 1 \\
\max Den(i, j) - \min Den(i, j) \leq L & \quad l(m) = i(j) - \frac{r}{2}, \ldots, i(j) + \frac{r}{2}
\end{align*}
\]

- **Combined Objectives**
  - linear summation of Min-Var, Lip-I and Lip-II objectives with specific coefficients

\[
\text{Minimize : } C_0 \cdot M + C_1 \cdot L_I + C_2 \cdot L_{II}
\]
  - add Lip-I and Lip-II constraints as well as

\[
M \leq W_{ij} \quad i, j = 0, \ldots, nr/w - 1
\]
## Computational Experience

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### Spatial Density Model

### Effective Density Model

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Comparison among LP methods on Min-Var and Lipschitz condition objectives (1)
### Computational Experience

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### Effective Density Model

Comparison among LP methods on Min-Var and Lipschitz condition objectives (2)

- LP with combined objective achieves the best comprehensive solutions
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- *Summary and Future Research*
Summary and Future Research

- Smoothness gap in existing fill methods
  - for the first time, we show the viability of gridless window analysis for both spatial density model and effective density model

- New smoothness criteria: local uniformity
  - three new relevant Lipschitz-like definitions of local density variation are proposed

- Ongoing research
  - extension of multi-level density analysis to measuring local uniformity w.r.t. other CMP models
  - improved methods for optimizing fill synthesis w.r.t. new local uniformity objectives