

Efficient Generation of Short and Fast Repeater Tree Topologies

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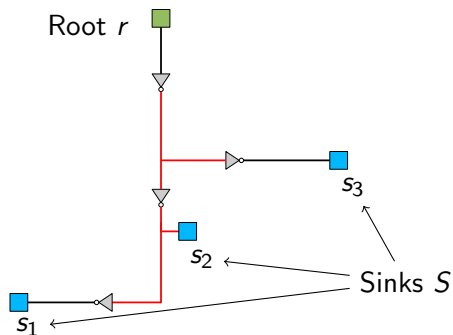
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Outline

- ▶ Repeater Tree Problem
- ▶ Delay Model
- ▶ Topology Construction Algorithm

The Repeater Tree Problem



- ▶ A signal has to be distributed from a source to a set of sinks.
- ▶ The **delay** on a source-sink path **increases**
 - ▶ **linearly in path length** (assuming ideal repeater insertion),
 - ▶ with every **bifurcation** on the path.

The Repeater Tree Problem

Objectives

- ▶ Minimize power consumption
- ▶ Minimize wiring
- ▶ Maximize worst slack σ_r , where

$$\sigma_r := \min_{s \in S} \{RAT_s - \text{signal_delay}(r, s)\}$$

The Repeater Tree Problem

Two-step Approach

First a repeater tree topology is constructed. Then repeaters are inserted in a second step (for example using a van Ginneken's style algorithm).

One-step Approach

Repeater insertion and topology generation are interleaved.

In this paper we focus on the topology generation in a two-step approach.

Previous Work

General Approaches to Topology Generation

- ▶ Minimum length rectilinear steiner tree
- ▶ Minimum spanning tree
- ▶ Shortest path trees

Problem-specific Approaches to Topology Generation

- ▶ C-Tree [Alpert et al., 2001]
- ▶ PRAB [Hu, Alpert, 2004]

Delay Estimation

- ▶ BELT [Alpert et al., 2004]

Our contribution

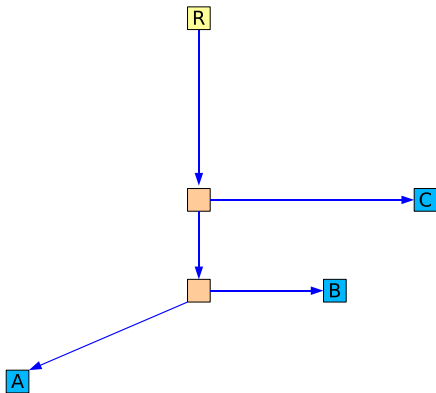
- ▶ A new delay-model for evaluating repeater tree topologies
- ▶ Theoretical bounds on the achievable slack
- ▶ A fast algorithm for topology construction considering our delay-model
- ▶ Optimality statements for our topology generation

Topology

A **topology** T is a directed tree rooted at r with $\delta^+(r) = 1$ and $\delta^+(u) = 2$ for all internal nodes u .

The set of leaves is a subset of S .

All internal nodes u are assigned placement coordinates $Pl(u)$.



Delay Model

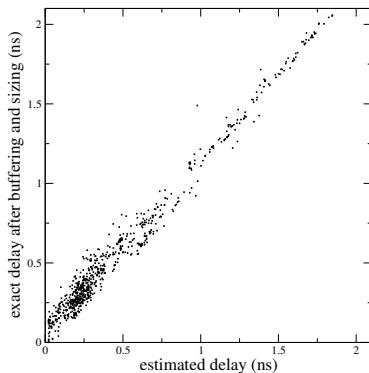
The delay from r to a sink s in a given topology is modeled as:

$$c_{node} \cdot (|E(T_{[r,s]})| - 1) + c_{wire} \sum_{(u,v) \in E(T_{[r,s]})} dist(Pl(u), Pl(v))$$

- ▶ c_{node} : Delay penalty for bifurcation
- ▶ c_{wire} : Delay per unit length
- ▶ Typical values are $c_{node} = 20$ ps and $c_{wire} = 220$ ps/mm.

Justification of Delay Model

Relation between critical path delays in our model (estimated delay) and with exact timing analysis after repeater insertion.



Bound on Wire Length

A lower bound on the wire length in our model is given by a minimum length rectilinear steiner tree (SMT).

Bound on Slack for Integer Values

Theorem 1

For $c_{wire} = 0$, $c_{node} = 1$ and integer values for AT_r and RAT_s for each $s \in S$ the maximum possible slack with respect to our delay model is:

$$- \left\lceil \log_2 \left(\sum_{s \in S} 2^{AT_r - RAT_s} \right) \right\rceil$$

Proof of Theorem 1

By Kraft's inequality there exists a rooted binary tree with n leaves at depth l_1, l_2, \dots, l_n if and only if

$$\sum_{i=1}^n 2^{-l_i} \leq 1$$

To realize a slack of at least σ we must find a topology in which $RAT_s - AT_r - d_s \geq \sigma$ holds for every sink s . The value d_s corresponds to the depth of sink s .

The maximum slack that can be realized is the largest integer σ_{max} that satisfies:

$$\sum_{s \in S} 2^{AT_r - RAT_s + \sigma_{max}} \leq 1$$

Bound on Slack

Theorem 2

The maximum possible slack σ_{\max} with respect to our delay model at root is at most:

$$-c_{node} \cdot \log_2 \left(\sum_{s \in S} 2^{-\left(\frac{RAT_s - c_{wire} \text{dist}(PI(r), PI(s))}{c_{node}} \right)} \right)$$

Sketch of Proof

Using Kraft's inequality and

$$RAT_s - AT_r - c_{wire} \text{dist}(PI(r), PI(s)) - c_{node} d_s \geq \sigma_{\max}$$

Improving the Upper Bound

The closed formula has two drawbacks:

- ▶ Integrality properties of the topology are neglected.
- ▶ Correct evaluation leads to numerical problems.

A better upper bound can be obtained algorithmically by using Huffman coding:

- ▶ No closed formula.
- ▶ Slightly better bounds.
- ▶ Numerical stable and loglinear runtime.

Using Huffman Coding

1. Set $\sigma_s = RAT_s - AT_r - c_{wire}dist(PI(r), PI(s))$ for all $s \in S$.
2. Order these values

$$\sigma_{s_1} \leq \sigma_{s_2} \leq \dots \leq \sigma_{s_n}$$

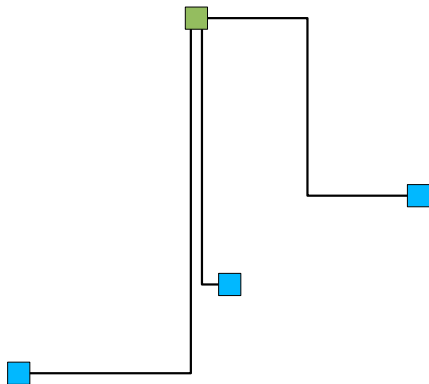
3. Replace the largest two $\sigma_{s_{n-1}}$ and σ_{s_n} by

$$-c_{node} + \min\{\sigma_{s_{n-1}}, \sigma_{s_n}\} = -c_{node} + \sigma_{s_{n-1}}$$

4. Go to 2.

Realization of the Maximum Slack

The maximum possible slack can be obtained by a shortest path tree:

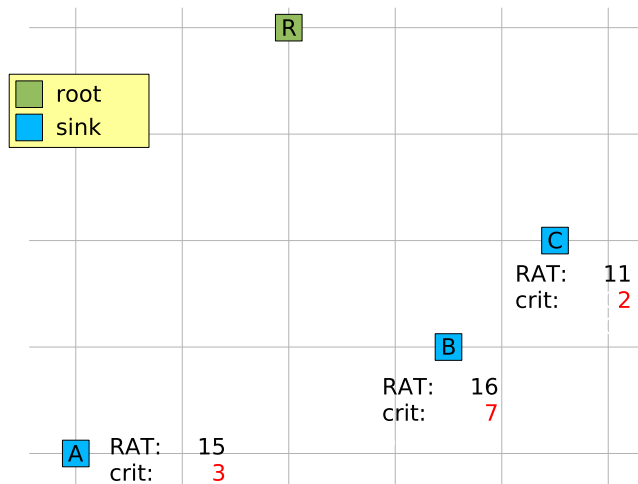


All distance delays are minimum: For each sink s , the distance part of the modeled delay attains the minimum possible value.

Topology Construction Algorithm

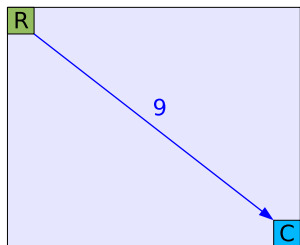
1. Sort sinks according to criticality (worst to best).
2. Start with a tree consisting of r and the first sink.
3. For each sink s , connect s to an edge of the tree, minimizing the cost function.

Example Problem Instance



Connect first sink

9 delay estimation



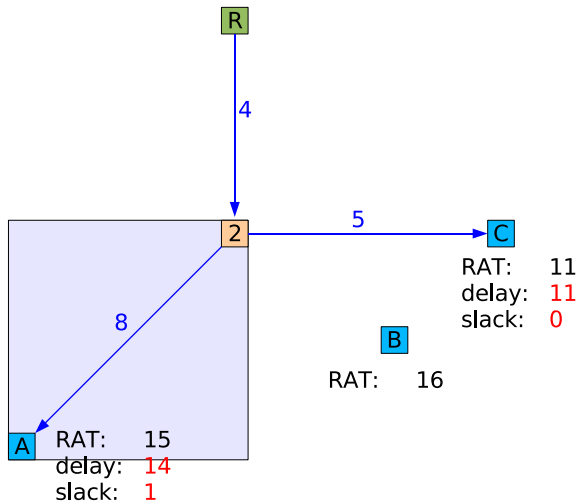
RAT: 11
delay: 9
slack: 2

B

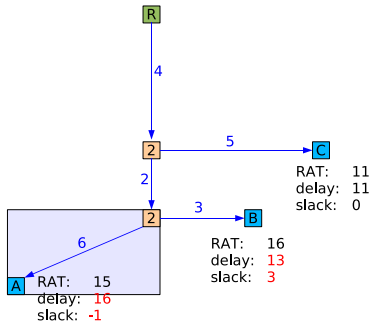
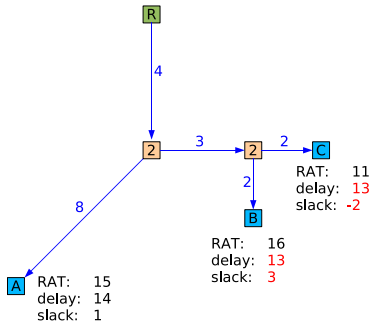
RAT: 16

A RAT: 15

Connect second sink



Connect third sink



Prim-Heuristic for Steiner Trees

Wire Length Minimization:

- ▶ Instead of choosing next critical sink:
- ▶ Choose sink, which is closest to the preliminary topology T' .
- ▶ Well known heuristic existing in many variants.

Hwang $\implies \frac{3}{2}$ -approximation algorithm for SMT.

Theorem 3

For $c_{wire} = 0$, $c_{node} = 1$ and integer values for RAT_s , $s \in S$, the algorithm generates a topology that realizes the maximum possible slack.

Proof.

Assume the sinks in $S' \subset S$ are already connected optimally in T' .
Let $s' \in S \setminus S'$.

- ▶ If all $s \in S'$ have the same slack $\sigma_{S'}$ in T' .
 - ▶ They are connected at maximum possible slack.
 - ▶ The best possible slack for the set $S' \cup s'$ equals $\sigma_{S'} + 1$.
 - ▶ s' can be connected to any existing edge in T' such that its slack is $\leq \sigma_{S'} + 1$.
- ▶ Otherwise s' can be connected to any non-critical edge.

Running Time

The running time is $O(|S|^2 \cdot \Psi)$, where Ψ is the running time of the cost function.

Handling Large Instances

- ▶ Pre-clustering if $|S| > 10\,000$
- ▶ Facility location approximation [Massberg, Vygen 2005]
- ▶ Runtime: $O(|S| \log |S|)$

Parameter Generation

Delay per nanometer

Insert repeaters in a 5 m long two-point net such that delay is minimized.

Delay per bifurcation

Insert a medium-sized repeater half-way between two repeaters of such a net.

Experimental Results

- ▶ 2.3 million instances with up to 10 000 sinks were taken from current 90nm designs.
- ▶ The slack minimizing cost function is compared against the slack bound (Huffman Coding).
- ▶ A length minimizing cost function is compared against a length bound.
- ▶ The topologies were computed in ≤ 50 seconds on a 2.6 GHz Opteron.

Results

# Sinks	# Instances	Wirelength Deviation (%)		Slack Deviation (ps)		Wirelength Deviation (%)		Slack Deviation (ps)	
		avg.	worst	avg.	worst	avg.	worst	avg.	worst
1	1547517	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	319759	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	165448	0.00	0.00	13.89	82.72	12.19	99.60	0.12	20.00
4	86377	0.16	19.65	23.72	312.98	10.93	190.27	0.27	40.00
5	44301	0.16	21.51	33.40	174.51	14.01	188.15	0.34	52.45
6	27854	0.28	23.84	41.92	118.27	14.38	268.06	1.04	52.93
7	20523	0.45	22.24	52.19	285.43	22.26	248.77	0.42	52.51
8	19300	0.44	30.73	64.01	332.29	19.39	268.49	2.08	69.13
9	11085	0.81	26.26	71.11	465.77	29.58	250.04	3.36	60.00
10	11942	0.74	28.68	76.46	367.39	23.61	296.47	1.45	54.87
11-20	38184	1.60	28.00	101.16	427.25	32.57	426.68	1.73	76.80
21-30	11104	3.20	30.80	144.27	520.00	35.86	805.45	2.51	84.18
31-50	8647	2.99	33.16	226.05	793.70	70.29	1091.17	6.55	161.81
51-100	6621	4.06	26.34	344.88	1486.06	105.90	1782.56	12.23	203.48
101-200	1863	5.82	16.91	606.26	2019.90	135.84	1498.34	19.78	351.25
201-500	824	6.22	24.00	920.37	3711.47	209.77	2127.34	26.91	304.92
501-1000	205	7.62	19.40	1686.15	3563.61	569.58	2242.49	48.57	257.65
> 1000	31	6.99	14.74	2929.08	7872.96	211.40	1124.99	17.78	89.88
Total	2321585	0.66	33.16	9.92	7872.96	19.35	2242.49	0.21	351.25
> 2 sinks	774068	1.31	33.16	50.69	7872.96	38.34	2242.49	1.08	351.25

Table: Deviation from known bounds, 90 nm

Thank you

